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AC-1624

M. Sc. (Sem. IV) Examination

April / May - 2015

PH (T) : 544 : Physics

(Group Theory & Quantum Field Theory)

Time : 3 Hours] [Total Marks : 70

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ M. SC. (SEM. IV)

Name of the Subject :

☛ PH (T) : 544 : PHYSICS

☛ Subject Code No. : 1 6 2 4 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

(2) Attempt all questions.
(3) Symbols used have their usual meaning.
(4) Figures to the right indicate marks.
(5) Assume data whenever necessary.

1 Attempt any two questions.

(i) (a) Define direct product of representation of a group. 3

(b) Explicitly show that if σ_u , σ_v, $C_4^2 \in C_{4v}$ group, then $C_4^2 \sigma_u = \sigma_v$. 4

(ii) (a) Define the character of a representation of a group and state the orthogonality theorem for characters. 3

(b) Show that the set of all non-singular matrices of order n form a group under matrix multiplication. 4

(iii) (a) What is meant by conjugate element of a group? What is the conjugate element to $m_y \in C_{4v}$. 3

(b) Explain the reducibility of representation of a group. What is meant by reducible representation and an irreducible representation of a group? 4

2 Attempt any two questions.

(i) (a) What is meant by a continuous group? Give an example. 3

(b) Consider a circle of radius a and let x measure the distance along the circumference. Let $f=f(x)$ and let $T(\varphi)$ stand for a rotation of the function f through an angle φ about an axis normal to the circle and passing through its centre. Find the generator for this transformation and express the operator $T(\varphi)$ in terms of the generators. 4

(ii) (a) What are the groups O(3) and SO(3)? Show that O(3) =SO(3) ⊗ (E, J). 3

(b) Consider a linear homogeneous transformation of two variable of the form 4

$$x' = a_{11}x + a_{12}y,$$
$$y' = a_{21}x + a_{22}y, \quad \text{with det } A = |a_{ij}| \neq 0$$

Show that the set of all such transformation is a four-parameter group.

(iii) (a) Express a general element of a Lie group with r continuous parameters in terms of the corresponding generators. 3

(b) Discuss the Lorentz group. 4

3 Attempt any two questions.

(i) (a) Define number, creation and annihilation operators for a scalar field. 3

(b) Discuss the Lagrangian formulation and quantization rules for a charged scalar field. 4

(ii) (a) Define vacuum state of a free scalar field. 3

(b) Define normal product operator and find $:aa^{\dagger} + aa^{\dagger}a:$. 4

(iii) (a) What is meant by Lagrangian density and Hamiltonian density functions? 3

(b) Write the free field Lagrangian density for the spin zero field and obtain the field equation corresponding to the Klein-Gordon equation. 4

4 Attempt any two questions.

(i) (a) Write the Feynman diagram with one loop correction in the case of electron-electron scattering with a static charge. 3

(b) Apply the Feynman rules for the above case and express the scattering amplitude. 4

(ii) (a) Discuss the rules for Feynman graphs in configuration space. 3

(b) Discuss in some details the idea of renormalization. 4

(iii) (a) Establish the anti-commutation relations for the Dirac field. 3

(b) Describe the initial and final state vectors for the scattering of a photon by an electron (Compton Scattering) and draw the corresponding Feynman diagrams and give the expression for the S-matrix elements. 4

5 Attempt any two questions.

(i) (a) Explain the main features of QCD as the theory for strong interaction. 3

(b) Discuss the Higg's mechanism with reference to spontaneous symmetry breaking. 4

(ii) (a) Express the interaction Lagrangian, $L_i = -g_i :[\bar{R}_e \varphi^{\dagger} L_e + \bar{L}_e \varphi R_e]:$ in terms of the physical fields ψ_e and Φ_0. 3

(b) Write the interaction Lagrangian and draw the Feynman diagrams for the process: (a) emission of a photon by an electron; (b) electron-positron annihilation. 4

(iii) (a) What are gauge transformations? Explain the gauge principle and gauge fields. 3

(b) Discuss the spontaneous breaking of global symmetry in Goldstone model. 4